

**THE ALTERNATIVE ANALYSIS: ITS BASIS, THEORY AND SOME APPLICATIONS**

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*Let  $N$  be the positive integers set. In this paper we prove that any mapping  $f: N \rightarrow A$ , with  $A \subset N$ , cannot be an injective one, i. e.  $(A \subset N) \Rightarrow \neg(A \sim N)$ . We use the theorem about of linear  $n$ -space basis, that is there does not exist any injective mapping of basis  $B_{n+1}$  into a basis  $B_n$ . In other words, we proved Euclidean Axiom 8: “The Whole is more than its own Part”; there are some proofs this theorem in our article.*

*Key words: Euclidean Axiom 8, natural numbers set, continuum-hypothesis, an injection, a sequence of natural numbers, a countable set, exact-permutation.*

**1. Introduce.** We use known mathematical texts in this report and we follow the Paul Cohen’s forecast about continuum-hypothesis (CH) [1, IV.13]: «A point of view which the author feels may eventually come to be accepted is that CH is obviously false». Linear independence (dependence) is the main concept of linear space  $E_n$ . “If there is a finite number of vector in the basis, the space is said to be finite dimensional and its dimension is equal to the number of vectors in its basis. Otherwise, it is infinite dimensional. For an infinite dimension space a basis usually means a sequence of elements  $x_1, x_2, \dots$  such that every  $x$  is uniquely expressible in the form  $x = \sum_{i=1}^{\infty} a_i x_i$  (meaning that the limit as  $n$  becomes infinite  $x - \sum_{i=1}^n a_i x_i$  is zero)”. [2, p. 27] The basis  $B_n$  of  $E_n$  contains  $n$  linearly independent vectors and the set of any  $n + 1$  vectors in  $E_n$  are linearly dependent and there does not exist any injective mapping of basis  $B_{n+1}$  on  $E_{n+1}$  into the basis  $B_n$  on  $E_n$ . Now we shall generalize those statements on infinite-dimensional linear space  $E_{\infty}$  easily. Let  $F(A, B) \triangleq \{f \mid f: A \rightarrow B\}$  be a set of all mappings [3, Sec. 8] from  $A$  in  $B$  (on  $B$ ). A mapping  $\varphi: A \rightarrow B$  is said to be injective one, or an injection, if either

$$a = q \text{ holds } \varphi(a) = \varphi(q), \text{ or } \varphi(a) \neq \varphi(q) \Rightarrow a \neq q. \tag{1}$$



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Instead of “the  $f$  is injective one” one speak [4, II.3.7] also, that “the  $f$  is one-to-one function” or “the  $f$  is 1–1–correspondence”. The mapping  $\varphi: A \rightarrow B$  of this kind that  $\varphi(A) = B$  is said to be “as mapping on” or “a surjective mapping”, or, more shortly, “a surjection”. Injective mapping  $\varphi: A \rightarrow B$  is named as bijective mapping or bijection [4, II.7] [5, I.6] if it is surjective too, i. e., when it is true (1) and  $\varphi(A) = B$ . In this case one speaks, that the sets  $A$  and  $B$  either are bijective, or they have equal power [5, I.9] and he writes  $A \sim B$ . The bijection  $\varphi: A \rightarrow A$  is named also a rearrangement, or a permutation, or a transformation of the set  $A$ . Let the symbols  $I(A, B)$ ,  $S(A, B)$  and  $B(A, B)$  designate the sets

of injections, surjections and bijections from  $A$  in  $B$  or, accordingly, into  $B$ . So at those notations we have the following equality

$$B(A, B) = I(A, B) \cap S(A, B). \tag{2}$$

Equality (2) shows that we must confirm both two properties of a mapping  $\varphi: A \rightarrow B$ : 1) the injectivity of  $\varphi$  and 2) its surjectivity  $\varphi(A) = B$ , before we can say that the mapping  $\varphi: A \rightarrow B$  is any bijection. Unfortunately, this requirement is ignored at any operating with the term (1–1)–correspondence either implicitly, or by default. Below we need a criterion of a

bijection of a mapping  $\varphi: A \rightarrow B$  being demonstrated in [6, Th.3.10], so everyone can prove that easily.

**Theorem 1.** Criterion of a bijection. The mapping  $\varphi: A \rightarrow B$  is bijective one if and only if for every splitting of the set  $A = \cup_i A_i$  into nonintersecting subsets  $A_i, i \in I \subset N$ , tree following conditions are fulfilled:

- (1)  $\forall i$  mapping  $\varphi_i: A_i \rightarrow B$  is any injection, where  $\varphi_i \triangleq \varphi|_{A_i}$ ,
- (2)  $\forall (i, j: i \neq j) B_i \cap B_j = \emptyset$ , where  $B_i = \varphi(A_i)$ ,
- (3)  $\cup_i B_i = B$ .

However, for examples, in 1950 H. Hasse wrote [7, I.1.1], that a mapping  $\varphi: N \rightarrow \{N \cup \{0\}\}$  “with  $\varphi(n) = n - 1$  is a substitute”. This sentence contradicts with Theorem 1 by means of uniqueness theorem for the set  $N$  existence.

**2. Galileo Galilei’s paradox.** Now to be followed to Peano [8, 3.1] we formulate the Axioms System for the set  $N$  with the equality relation ( $x = x, a \neq b$ ):

- (P1)  $0 \in N$ .
- (P2)  $\forall x \in N, \exists x' \in N$ , then  $x'$  is said to be “immediately follows to  $x$ ”.
- (P3)  $\forall x \in N x' \neq 0$ .
- (P4)  $\forall y \in N \setminus \{0\} \exists! z \in N: (z)' = y$ .
- (P5) If any attribute  $Q(x)$  is defined  $\forall x \in N$ , and if
  - (I)  $Q(0) = \text{truth}$  and
  - (II)  $\forall x \in N Q(x) = \text{truth}$  holds  $Q(x') = \text{truth}$ , therefore,
  - (III)  $\forall y \in N Q(y) = \text{truth}$ .

Axiom (P5) affirms the principle of mathematical induction. Below we use this mathematical induction technique every often. Galileo Galilei discovered on 1638 [9, c. 140–146; 10, IV.32] the paradox which is  $\forall a \in N \exists b = a^2 \in N$ . G. Galilei has said about this as following: “The notions both either equality or inequality can be used only for finite qualities and not be applied for the infinite ones”.

However, on the boundary of XIX and XX centuries there was appeared far-reaching with mistakes generalization of G. Galilei’s opinion about his paradox. S. Kleene write down [10, IV.32] this point of view as following: “... it is possible to establish 1–1–correspondence between squares of the positive integers numbers and itself integers, that conflicts with Euclidian Axiom 8 [11], according to which whole more than any of its own parts...”

Now we shall give a new interpretation of G. Galilei’s paradox. To be exact yet we prove the mapping  $\varphi: N \rightarrow N$ , with  $\varphi(n) = n^2$ , is not realizable on the all set  $N$ . We shall consider the square  $I_n \times I_n$ , where  $I_n \triangleq (0, 1, 2, \dots, n)$ . Let now  $S(n)$  be the following statement: “The  $\varphi(n) = n^2$  is not realizable on the all  $I_n$  “. We prove  $\forall n \in N$  the statement  $S(n)$  by mathematical induction technique.

- (I) Let  $n=2$ , so  $I_2 = (0, 1, 2)$  and  $\varphi(I_2) = (0, 1, 4) \not\subseteq I_2 = (0, 1, 2)$ . Then  $S(2) = \text{truth}$ .
  - (II.1) Let  $n=k$  and  $S(k) = \text{truth}$  by induction.
  - (II.2) Let  $n=k+1$ . Then  $\varphi(I_{k+1}) = \varphi(I_k) \cup \varphi(k+1)$ . Since  $\varphi(k+1) = (k+1)^2$  and with (II.1)  $\varphi(I_{k+1}) \not\subseteq I_{k+1}$ . Therefore,
  - (III)  $\forall n \in N, n > 1, S(n) = \text{truth}$ .
- So we proved Euclidean Axiom 8 at the first time.

**3.** Stephen C. Kleene had used [12, I, 1,2] the Cantor diagonal method for the proof an uncountability of all natural sequences sets. Namely, he has chosen the countable subset of natural sequences  $(f_j^*(k))$  from the set  $(f_i(k))_{i,k=0}^{l,k=\infty}$  of all ones. Haskell Curry has described the similar example as Rishar’s paradox. Farther, S. Kleene wrote the sequences  $(f_j^*(k))$  in the infinite-dimensional square matrix

$$H \triangleq \begin{pmatrix} f_0^*(0) & f_0^*(1) & f_0^*(2) & \dots \\ f_1^*(0) & f_1^*(1) & f_1^*(2) & \dots \\ f_2^*(0) & f_2^*(1) & f_2^*(2) & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}. \quad (3)$$

Further S. Kleene has defined one more sequence  $f^*(k)$  under the formula  $f^*(k) \triangleq f_k^*(k) + 1$ . Matrix  $H$  of countable subsets of the set  $(f_j^*(k))$  of all sequences. The sequence  $f^*(k)$  was not inscribed into matrix (3) because this list-matrix  $H$  was already formed as completely and so finely. And consequently Stephen C. Kleene has formulated the following statement: “The set of all number natural sequences is not countable one.” Really, S. Kleene has proved much greater. He proved that there not exist any bijection between the set  $H$  of sequences  $f_i^*(k)$  and set  $H^* \triangleq H \cup \{f^*(k)\}$ . We write below the details of some proofs of this statement.

Let  $B_{n+1} \triangleq (b_0, b_1, \dots, b_n)$  and  $H_n \triangleq (h_0, h_1, \dots, h_{n-1})$  bebases of two linear spaces  $E_{n+1}$  and  $E_n$ , accordingly. We shall use the mathematical induction technique for proving the following statement.

**Theorem 2.**

$$\forall n \in \mathbb{N} \exists (i, j, m): i \neq j, i + 1, j + 1, m \leq n: \forall (\varphi: B_{n+1} \rightarrow H_n) \Rightarrow \varphi(b_i) = \varphi(b_j) = h_m.$$

(I) Let  $n=1$ , then  $B_{n+1} \equiv B_2 = (b_0, b_1)$  and  $H_n \equiv H_1 = (h_0)$ .

Now we have following equalities:  $\varphi(b_0) = \varphi(b_1) = h_0$ , so Th. 2 = truth.

(II.1) Let  $n=k$  and Th. 2 = truth, by a condition of induction.

(II.2) Let  $n=k+1$ .

Then  $B_{n+1} \equiv B_{k+2} = (b_0, b_1, \dots, b_{k+1})$ ,  $H_n \equiv H_{k+1} = (h_0, h_1, \dots, h_k)$ . Now we have  $\varphi(B_{k+2}) = \varphi(B_{k+1} \cup \{b_{k+1}\}) = \varphi(B_{k+1}) \cup \varphi(\{b_{k+1}\}) \subseteq H_{k+1}$ . Thus  $\exists m, m \leq k + 1: \varphi(b_{k+1}) = h_m \in H_{k+1}$ . At the second hand, by virtue  $\varphi(B_{k+1}) \subseteq H_{k+1}, \exists l, l < k + 1: \varphi(b_l) = h_m$ .

Then we have

$\varphi(b_l) = h_m = \varphi(b_{k+1})$ . That is Th. 2 = truth in this case too. Therefore,

(III)  $\forall n \in \mathbb{N}, n > 1, \text{Th. 2} = \text{truth}$ .

So we proved Euclidean Axiom 8 at the second time.

**4. Cantor diagonal method and Euclidean Axiom 8.** Now we return to matrix (3). Let in (3) matrix  $H$  be identity matrix  $HE$

$$HE \triangleq \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & 0 & \dots \\ & & \dots & & & \\ 0 & 0 & \dots & 0 & 1 & 0 & \dots \\ & & & & & \dots \end{pmatrix}.$$

The lines of matrix  $HE$  are canonic natural sequences  $f_i \triangleq (\delta_i^k: k = \overline{0, \infty}), i = \overline{0, \infty}$ , here  $\delta_i^k$  – is L. Kronecker symbol. Those sequences make up a basis  $B(F)$  of infinite-dimensional space  $F$  that is the set of all natural sequences  $(f_i(k)_{k=0}^\infty)_{i=0}^\infty, i, k \in \mathbb{N}$  [2, p. 349]. For example, if  $f^* \triangleq (0 \ 1 \ 1 \ 0 \ \dots)$ , then  $f^* = f_1 + f_2$ . Now we identifier every element  $f_k$  of basis  $B(F)$  with the corresponding natural number  $k$  by means following bijection  $\varphi: \mathbb{N} \rightarrow B(F)$ ,

$$\varphi(k) = f_k. \quad (4)$$

Let  $F^* \triangleq B(F) \cup \{f^*\}$ . Now we shall prove the following main statement.

**Теорема 3.** There is not mapping  $\psi: F^* \rightarrow B(F)$  that is an injective one.

(1)  $\exists m: \psi(f^*) = b_m$ . (2) Yet the mapping  $\psi: B(F) \rightarrow B(F) \setminus \{b_m\}$  is not injective by virtue of both Theorem 1 and Theorem 2.

Theorem 3 is equivalent by virtue of the bijection (4) with Euclidian Axiom 8 in connection with the set  $N$  of all natural numbers in following form:

$$N \subset N \cup \{a, a \notin N\} \Rightarrow \neg(N \cup \{a\} \sim N).$$

The second example is more instructive: The bijective mapping exists by virtue of the bijection (4) between the set of all even natural numbers and the set, for example, those and only those vectors of the basis  $B(F)$ , which have even indexes. The legend about an existence of any bijective correspondence between of natural numbers set and the set of even numbers was constructed on any historical misunderstanding. Thus we proved Euclidean Axiom 8 once again.

1. **The anti-cyclic permutation.** In this item we use essentially the Axiom of Choice [1, IV.9], [13,0.23–0.26] and a notion of exact-permutation in our proofs. If  $f \in F(A, A)$  and  $H \subset A$ , then not always  $f(H) \neq H$ . A set  $B_{ex}(A, A)$  of all *exact rearrangements of a set A* (*exact-permutation* or anti-cyclic permutation, a rearrangements without cycles [6, Chap. 3]) we define by the following equality

$$B_{ex}(A, A) \triangleq \{f: f \in B(A, A)\} \& f(H) = H \Rightarrow H = A. \quad (5)$$

For example, the bijection  $f \in B_{ex}(I, I)$ ,  $I \triangleq [0, 1]$ , can be defined with following formula:

$$f(x) \triangleq \begin{cases} x + h, & \text{if } 0 \leq x \leq 1 - h, \\ 1 - x, & \text{if } 1 - h \leq x \leq 1, \end{cases}$$

where  $h < 0,5$  and  $h$  is a transcendental number.

The graph of this function in the obvious image testifies, that the given function  $f$  satisfies to a condition (5). It is easy to write down exact rearrangements on set of natural numbers but what would not be the first element in image at a mapping  $f \in B_{ex}(N, N)$ , the last element there cannot be, that is obvious, by virtue of the potential nature of set  $N$ .

Now we shall choose a pair  $\{a, \psi\}$ ,  $a \in A$  and  $\psi \in B_{ex}(A, A)$  by means of the Axiom of a choice [13, 0.23–0.24] from sets  $A$  and  $B_{ex}(N, N)$ . Further we define with the help of the pair  $\{a, \psi\}$  following sequence of investments of subsets of set  $A$ , named ([3, Sec. 14], [13, 0.9; 0.23]) as a chain (a chain on an investment):

$$\{a\} \subset \{a, b\} \subset \{a, b, c\} \subset \{a, b, c, \dots, p\} \subset \{a, b, c, \dots, p, q\} \subset \dots \subset A, \quad (6)$$

where  $b = \psi(a)$ ,  $c = \psi(b)$ ,  $d = \psi(c)$ ,  $\dots$ ,  $q = \psi(p)$ ,  $\dots$ .

We shall name a chain (6) by *a-chain* on its first element [6, Гл. 3]. Let's emphasize, that in (6)  $\forall Y \subset A \psi^{-1}(a) \notin Y$ , in general case. If  $Q$  and  $P$ ,  $P \subset Q \subset A$ , are the neighboring elements of *a-chain* (6), then  $Q = P \cup \{q\} \triangleq P^+$ , where  $\psi(p) = q$  and  $q$  is, accordingly, the greatest element in  $Q$  in the sense that,  $Q \setminus P = \{q\}$ . The last property defines *discrete character* of a chain (6). Therefore, it is obvious, that the set of elements of *a-chain* (6) is *quite ordered* set, i. e., its each non trivial subset has *the least element*. We shall name *a-chain* (6) a *full one*, meaning two its properties noted above. If  $\xi \in B_{ex}(A, A)$ , then, at the first,  $\xi(a) = g \neq a$ , and, secondly, the mapping  $\xi$  will transform *the initial full a-chain* (6) into some *full g-chain* which we shall designate as symbol  $P(g, \xi, A)$ . Thus, equality  $Q = P^+$  is equivalent to equality  $Q = P \cup \{q\}$ , hence, we have following equalities:

$$\xi(P^+) = \xi(P \cup \{q\}) = \xi(P) \cup \xi(\{q\}) = \xi(Q).$$

Order  $\nu$  on set  $A$  is defined by pair  $\{a, \psi\}$  as follows. If  $h \triangleq \psi^{-1}(a)$ , then  $\forall q \in A \setminus h$  we accept  $q \prec h$  and  $q \prec \psi(p)$ . Besides if for elements  $Q$  and  $R$  of *a-chain* (6)  $\exists H$  such, that  $Q \subset H \subset R$ , i. e.  $R \neq Q^+$ , then for  $\forall q \in Q$  and  $\forall s \in R \setminus Q^+$  we assume  $q \prec s$ . So that  $a \prec b$ ,  $b \prec c$ ,

$c \succ d, \dots$ , where, as in (6),  $b = \psi(a), c = \psi(b), d = \psi(c), \dots, q = \psi(p), \dots$ . By virtue of (6) order  $\nu$  is both *full* and *discrete*. Set  $A$  with the order  $\nu$  we shall designate as  $(A, \nu)$ .

**Remark 1.** If the element  $\psi^{-1}(a)$  exists, then we can define a decreasing chain on an investment

$$\begin{aligned} A \supset A \setminus \{a\} \supset A \setminus \{a, b\} \supset \{a, b, c\} \supset \dots \\ \supset A \setminus \{a, b, c, \dots, p\} \supset A \setminus \{a, b, c, \dots, p, q\} \supset \dots \supset \psi^{-1}(a) \end{aligned} \quad (7)$$

by means of a full chain (6) by obvious image. We name a chain (7) as dual-chain for  $a$ -chain (6).

**Theorem 4.**  $\forall (y, Y): y \in Y \subseteq A$  there exists  $\phi, \phi \in B(A, A)$ , such that  $y$ -chain  $P(y, \phi, A) \ni Y$ .

**Proof** Let  $Z \triangleq A \setminus Y, \sigma \in B_{ex}(Y, Y)$  and  $\tau \in B_{ex}(Z, Z)$ . Thus we have  $y$ -chain  $P(y, \sigma, Y)$ . Let  $Y \cup \{\tau\} \triangleq Y^+, \tau \in B_{ex}(Z, Z)$ . Now we construct the  $\phi \in B(A, A)$  as following: 1)  $\phi|_Y \triangleq \sigma$  and 2)  $\phi|_Z \triangleq \tau$ .

Using both Theorem 1 and Theorem 4 we can construct new proof of Euclid Axiom 8. Let  $(A, \nu)$  be a set  $A$  with order  $\nu$  from it.4. Further let  $\mathbf{P}(A)$  be a set of all subsets of set  $A$ . Now we consider the class  $\mathfrak{R}(\mathbf{P}(A))$  containing all elements of the set  $\mathbf{P}(A)$ , the set of all  $a$ -chains and the set of all dual-chains. Now everyone can easily prove the following statement.

**Theorem 5.** Any chain  $V$ , being either an own part of either some full  $q$ -chain or of the dual-chain is a well ordered one, i.e. this chain  $V$  has the least element.

**6. The quite ordered sets and injective mappings.** Now we prove Euclid Axiom 8 in the form of the following theorem.

**Теорема 6.** Let  $B \subset A$  and  $\varphi \in F(A, B)$ , then such pair  $(a, q)$  of elements  $a$  and  $q$  exists into the set  $A$ , that

$$\varphi(a) = \varphi(q) \text{ with } a \neq q. \quad (8)$$

**Proof** Let  $\varphi(A) = B$ . It will not break a generality of our reasoning. Now we shall assume opposite (8), i. e.,  $\varphi \in I(A, B) \neq \emptyset$  at  $B \subset A$  and let further  $H \triangleq A \setminus B$ , then  $H \cap B = \emptyset$ . Now we have a circuit of equalities in those designations:

$$B = \varphi(A) = \varphi(B \cup H) = \varphi(B) \cup \varphi(H).$$

Therefore,  $\varphi(B) \subseteq B$  and  $\varphi(H) \subseteq B$ . An inclusion  $\varphi(H) \subseteq B$  means, that  $\forall h \in H$  and  $b \in B$  exists such that  $\varphi(h) = b$ . On the other hand, if  $\varphi(B) = B$ , then an element  $g \in B$  exists for every  $b \in B$  by virtue of  $\varphi \in I(A, B)$  such, that  $\varphi(g) = b = \varphi(h)$  at  $g \neq h$ , since  $B \cap H = \emptyset$ . This proves the condition (8). In case  $\varphi(B) = B_1 \subset B$  and  $H_1 \triangleq B \setminus B_1$ , then  $\varphi(B) = \varphi(B_1 \cup H_1) = \varphi(B_1) \cup \varphi(H_1)$ . Therefore,  $\varphi(B_1) \subseteq B_1$  and  $\varphi(H_1) \subseteq B_1$ . As well as above we prove, that either  $\varphi(B_1) = B_1$  and the condition (8) is proved, or  $\varphi(B_1) = B_2 \subset B_1, H_2 \triangleq B_1 \setminus B_2$  and so on. Thus, we shall receive following a decreasing sequence  $\mathbf{Z}$  of investments:  $A \supset B \supset B_1 \supset B_2 \supset \dots \supset B_i \supset \dots$ . Here  $i \in J$  and  $J$  is some set of indexes corresponding constructed chain  $\mathbf{Z}$ . This chain  $\mathbf{Z}$  contains by virtue either of Theorems 4 and 5 or Kuratowski's lemma [13, 0.25.(d)] in some maximal chain. Therefore, the chain  $\mathbf{Z}$  has any least element in this case. It means that there  $\exists k \in J: \varphi(B_{k-1}) = B_k$  and  $\varphi(B_k) = B_k \subset B_{k-1}$ . If as above,  $H_k \triangleq B_{k-1} \setminus B_k \neq \emptyset$ , then  $\varphi(B_{k-1}) = \varphi(B_k \cup H_k) = \varphi(B_k) \cup \varphi(H_k) = B_k$ . Therefore,  $\varphi(B_k) = B_k$  and  $\varphi(H_k) \subseteq B_k$  at  $B_k \cap H_k = \emptyset$  by virtue of the subset  $H_k$  choice. It means that the conclusion (8) of Theorem 6 is proved. Consequently, we prove Euclidean Axiom 8 at the fourth time in this paper.

Theorem 6 has following the canonical brief form:  $B \subset A \Rightarrow \neg A \sim B$ .

In particular,  $q \notin A \Leftrightarrow \neg (A \sim (A \cup \{q\}))$ , and we speak about this so: concerning equivalence the family of infinite sets (as well as finite sets) divides on classes “to within an element”. This result, obviously in turn, opens a new way of research a continuum-hypothesis [1]. We have written down below without the proofs only two of statements which of them is equivalent to Theorem 6 and they both as well as Theorems 1-6 are obvious to the finite sets.

**Statement 1.** If  $B \subset A$  and  $\varphi \in I(A, B)$ , then  $\exists \xi \in (A, B): \xi|_B = \varphi$ .

**Statement 2.**  $A \sim B \Leftrightarrow (B_{ex}(B, B) \sim B_{ex}(A, A) \Leftrightarrow (B(A, A) \sim B(B, A)))$ .

The second part of the Statement 2, namely the equivalence

$$B_{ex}(B, B) \sim B_{ex}(A, A) \Leftrightarrow (B(A, A) \sim B(B, A)),$$

characterizes the property for the equivalent sets to be indiscernible in the functional altitude.

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## ИСПОЛЬЗОВАНИЕ ОНТОЛОГИЧЕСКОГО ПОДХОДА В СИСТЕМЕ УПРАВЛЕНИЯ ИНТЕЛЛЕКТУАЛЬНЫМ КАПИТАЛОМ ОРГАНИЗАЦИИ

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*Описывается развитие информационной системы для автоматизации обслуживания и ремонта измерительных приборов и нагревающейся автоматики и измерения теплоэлектростанции Норильска. Особенность в использовании онтологического подхода к построению системы интеллектуальной поддержки. Согласно результатам экспериментального использования системы, удалось оценить рентабельность и возможные области для дальнейшего развития.*

*Ключевые слова: Обслуживание и Восстановление Оборудования, Система планирования ресурсов предприятия, Система Управления знаниями.*